

Chattering-free Higher Order Sliding Mode and Unscented Kalman Filter for Blood Glucose Level Control

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Abstract – A new hybrid scheme is proposed for the blood glucose (BG) regulatory system in type-1 diabetes mellitus (T1DM) patients. Some of hybrid glucose-insulin controllers have combined the artificial intelligence such as fuzzy logic and metaheuristic algorithm with well-known Palumbo model. However, most of those approaches have focused on the glucose reference tracking, and the glucose sensor errors as well as various unmeasurable disturbances have not been well studied. In this paper, a hybrid method based on higher order sliding mode (HoSM) and unscented Kalman filter (UKF) has been presented. The high performance of the proposed hybrid scheme for desired BG level tracking and chattering reduction in presence of daily glucose level disturbances has been verified.

Keywords-component; Hybrid; Higher order Sliding Mode (HoSM); Unscented Kalman Filter (UKF); Chattering;

I. INTRODUCTION

Many papers have addressed that Continuous glucose monitoring (CGM) is a vital factor in randomized trials to improve glucose control in patients with type-1 diabetes mellitus (T1D) [1-4]. Mathematical models have become of great importance in the context of diabetes treatment planning. Some of the most important useful models that have been proposed for glucose-insulin regulatory system include the Bergman minimal model is presented in [5], the Sorensen model in [6], the Kuang model in [7], the Dalaman model in [8-9] and the Liu-Tang model [10]. A novel hybrid identification method has been introduced to model T1D patients and to effectively recover the unmeasured input signals such food, physical activity, emotions as well as sensor errors, [11]. During the last decade, more attention has been to delay differential equation (DDE) because of their compliance with diagnostic testing of patients [12-13]. The advantage of the DDE model is that the delay of insulin effects is considered as the time interval between BG level rise and insulin injection time. Some studies have shown that the artificial pancreas (AP) systems can be able to control blood glucose concentrations (BGC) [14]–[16].

Glucagon is the natural secretory product of pancreatic alpha cells in the body and is normally released in response to

hypoglycemia. Some other papers have shown that the issue of avoiding hypoglycemic events by incorporating a second pump that delivers glucagon [17-20]. A multiple model probabilistic predictive controller (MMPPC) has been assessed on T1D patients that do not require meal announcement [21].

By modifying the pancreatic insulin secretion based on the Liu-Tang model, the model could be transformed to describe type-1 diabetes mellitus (T1DM) [22]. Using adaptive neuro-fuzzy inference system (ANFIS) which combines the artificial neural network (ANN) adaptive approach and the fuzzy inference system (FIS), the blood glucose system will be regulated for a T1D patient. Also, the FIS structure was constructed using subtractive clustering approach [22]. Classical and traditional ANFIS based methodologies have some intrinsic problems that in presence of input disturbances such as food intake and physical activity often yields chattering. In [23], by switching on Bergman model and setting appropriate parameters and overshoot time, the suggested proportional-integral-derivative (PID) controller will be able to reach the BG level into a normal reference level. Controllers based on induced L2-norm minimization of glucose-insulin system [25], improved PID [25], fuzzy logic [26] and H_∞ robust control approach [27] have been studied and tested in clinical trials. However, most of the previously mentioned studies have focused on error reduction of the glucose reference tracking, and the qualitative of this tracking in T1D patients such as chattering reduction has not been well studied.

Palumbo and his colleagues in [28] have proposed one of the complete control methods in the field of glucose-insulin regulatory system based on DDE model. To overcome some drawbacks, a hybrid approach of the fuzzy logic controller and the genetic algorithm as an optimization tool called hybrid method was proposed to control the BG level [29]. Until now, only a few studies have addressed the chattering phenomena of real-time insulin infusion and its complications. Higher order sliding mode (HoSM) control techniques, such as super-twisting algorithm, were successfully used to eliminate the effect of chattering and obtain continuous control [30-32]. The

primary drawback to almost all traditional insulin-glucose regulatory approaches is that they are not able to recover or to systematically account for the various disturbances that affect T1D patients. These various unmeasured disturbances such as food, physical activity, emotions and actuator errors are main sources of the glucose fluctuating and cause the chattering phenomena in the insulin infusion rate.

Extended Kalman filter (EKF) and unscented Kalman filter are two conventional methodologies for nonlinear estimation. Through linearization, the extended Kalman filter (EKF) has been used for state estimation of nonlinear dynamic systems [33]. The EKF has shown that is computationally expensive and has some difficulties to implement in the estimation problems [33]. The UKF addresses the mean and covariance flaws from nonlinear terms utilizing a deterministic sigma points. Then the state distribution is represented by a Gaussian Random Variable (GRV), and is calculated using a minimal set of the selected sample points [34]. In contrast of the EKF, no explicit calculation of Jacobean matrix is necessary for the UKF estimation algorithm. In this paper, a hybrid scheme using the UKF and the HoSM controller has been suggested to overcome these drawbacks.

The rest of the paper is organized as follows. The Palumbo mathematical model and his nonlinear control for glucose-insulin regulatory system is reviewed in Section II. Section III describes the hybrid control approach. In Section IV, the proposed chattering-free hybrid HoSM and UKF (HoSM-UKF) method is explained. Simulation of glucose-insulin system is presented in Section V. Finally, the conclusions of this paper are discussed in Section VI.

II. THE PALUMBO DDE MATHEMATICAL MODEL

The DDE mathematical model for glucose-insulin regulatory system is considered as follows [12]:

$$\dot{G}(t) = -K_{xgi}I(t)G(t) + \frac{T_{gh}}{V_g} \quad (1)$$

$$\dot{I}(t) = -K_{xi}I(t) + \frac{T_{igmax}}{V_i}f(G(t - \tau_g)) + u(t) + d_I(t) \quad (2)$$

$$f(G) = \frac{(\frac{G}{G^*})^\gamma}{1 + (\frac{G}{G^*})^\gamma} \quad (3)$$

Where the delayed pancreatic insulin delivery rate is modeled as the nonlinear function $f(G)$. $u(t)$ and $d_I(t)$ are considered as a control input and a unavoidable actuator disturbance, respectively. The purpose of a proposed control law is the reduction of plasma glucose concentration above the normal level and keeps this value at the normal level. That is possible through intravenous insulin infusion. Palumbo delayed nonlinear model parameters are described in follows [12]:

- $G(t)$ in [mM] denotes the plasma glucose.
- $I(t)$ in [pM] denotes the plasma insulin.
- K_{xgi} in [$\text{min}^{-1}\text{pM}^{-1}$] is the rate of glucose uptake by insulin dependent tissues per pM of plasma insulin concentration.

- T_{gh} in [min^{-1} (mmol/kgBW)] is the net balance between hepatic glucose output and insulin independent zero-order glucose tissue uptake.
- V_g in [L/kgBW] is the apparent glucose distribution volume.
- K_{xi} in [min^{-1}] is the apparent first order disappearance rate constant for insulin.
- T_{igmax} in [min^{-1} (pmol/kgBW)] is the maximal rate of second phase insulin release.
- V_i in [L/kgBW] is the apparent insulin distribution volume.
- τ_g in [min] is the apparent delay with which the pancreas varies secondary insulin release in response to varying plasma glucose concentrations.
- $u(t)$ in [pM/min] is the Insulin Injection per minutes.
- G^* correspond to the glucose at which the insulin release is half its maximal rate.
- γ denotes the progressivity with which the pancreas reacts to circulating glucose concentrations.

As mentioned before $d_I(t)$ in (2) denotes the actuator error. This disturbance is assumed measurable and generally considered as a sinusoidal variable [12].

$$d_I(t) = a_I \sin(w_I t). \quad t > 0 \quad (4)$$

In order to consider the glucose disturbances such as meal intake at $t = t_m$, another disturbance term $d_G(t)$ is considered in (1) as below:

$$\dot{G}(t) = -K_{xgi}I(t)G(t) + \frac{T_{gh}}{V_g} + d_G(t) \quad (5)$$

$$d_G(t) = \alpha \exp(-\beta(t - t_m)). \quad t > t_m \quad (6)$$

Palumbo et al. after presenting the above mentioned nonlinear model of glucose-insulin, provided a glucose-insulin regulatory system based on feedback linearization to eliminate the nonlinear terms [13]. This control approach based on the feedback linearization is as follow:

$$u(t) = \frac{S(G(t), I(t), G(t - \tau_g)) - w(t)}{K_{xgi}G(t)} \quad (7)$$

$$S(G(t), I(t), G(t - \tau_g)) = -K_{xgi}I(t) \left(-K_{xgi}I(t)G(t) + \frac{T_{gh}}{V_g} \right) \quad (8)$$

$$-K_{xgi}G(t) \left(-K_{xi}I(t) \frac{T_{igmax}}{V_i} f(G(t - \tau_g)) \right)$$

$$w(t) = \ddot{G}_{ref}(t) + Ke(t) \quad (9)$$

$$e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = q(t) - q_{ref}(t) \quad (10)$$

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} = \begin{bmatrix} G(t) \\ -K_{xgi}I(t)G(t) + \frac{T_{gh}}{V_g} \end{bmatrix} \quad (11)$$

$$q_{ref}(t) = [q(t) \quad \dot{q}_{ref}(t)]^T \quad (12)$$

Where $G_{ref}(t)$ denotes the glucose reference level and in (9), vector $K \in \mathbb{R}^{1 \times 2}$ is determined in such a way that the matrix H in (13) has the eigenvalues located on the left side of the imaginary axis.

$$H = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} K \quad (13)$$

This method is based on trial and error to find the desired location of closed-loop poles. To overcome this problem, a hybrid of the fuzzy logic controller and feedback linearization has been suggested [29].

III. PROPOSED HIGHER ORDER SLIDING MODE AND UNSCENTED KALMAN FILTER (HoSM-UKF) CONTROL

A. Higher order sliding mode (HoSM)

The collection of actuator errors, sensor errors as well as various unmeasurable disturbances of the glucose-insulin regulatory mechanism is the basic requirement to obtain an accurate hybrid model. Considering all above mentioned errors will cause some fluctuations in the regulatory system. Therefore, such input control fluctuations must be strictly avoided in biological systems which can cause damage to the patient body. To solve the contradiction between the reference glucose tracking and smooth insulin infusion rate, the HoSMCs were proposed in Bergman model [30]. Although the meal disturbances and dynamics of the insulin pump were considered, but the effect of measurement noise was not considered [30]. The Palumbo model does not consider other unmeasured input signals such as physical activities and emotions which is a common relevant problem to the traditional controllers that based on DDE models. In the diabetes context, an important problem is about the not easily measurable inputs such as glucose or body insulin, while the conventional glucose-insulin control techniques don't consider or be able to handle such disturbances. In our proposed approach, to show the complete dynamics of the glucose-insulin regulatory system, the $d_G(t)$ in (5) is considered as below:

$$d_G(t) = a \exp(-\beta(t - t_m)) + a_G \sin(w_G t) + \gamma \Pi\left(\frac{t - t_0}{T}\right) + \delta w(t) \quad (14)$$

Where the first term is considered the meal intake, the normal physical activity is defined by a sinusoidal function, the constant physical activity is considered as a rectangular function with duration T and $w(t)$ considered as a Gaussian white random process. And also the measurement noise $v(t)$ must be considered for the glucose measurement as follows:

$$Z(t) = G(t) + v(t) \quad (15)$$

The tracking error is defined as the difference between the glucose value and the glucose reference value in T1D patient's blood as in (16), (17).

$$e(t) = G(t) - G_{ref}(t) \quad (16)$$

$$\dot{e}(t) = \dot{G}(t) - \dot{G}_{ref}(t) \quad (17)$$

A suitable controller $u(t)$ would be designed such that e_G and $\dot{e}(t)$ goes to zero in the presence of all noises and disturbances. At first, the relative degree of the system for HoSM control should be calculated. For dynamic equation (1), relative degree r means the first appearance of control $u(t)$ explicitly in the r^{th} successive differentiation of tracking error e_G . Considering (1) without disturbances and noises, the control variable appears in the derivatives after the second differentiation, i.e.

$$\begin{aligned} \ddot{G}(t) &= -K_{xgi}I(t)G(t) - K_{xgi}I(t)\dot{G}(t) \\ &= -K_{xgi} \left[-K_{xi}I(t) + \frac{T_{igmax}}{V_i} f(G(t - \tau_g)) + u(t) \right] G(t) - K_{xgi}I(t)\dot{G}(t) \end{aligned} \quad (18)$$

The above equation could be considered as:

$$\ddot{G}(t) = \Phi(G, I, t) - K_{xgi}G(t)u(t) \quad (19)$$

Where

$$\begin{aligned} \Phi(G, I, t) &= K_{xgi}K_{xi}I(t)G(t) \\ &+ \frac{T_{igmax}}{V_i} f(G(t - \tau_g)) G(t) - K_{xgi}I(t)\dot{G}(t) \end{aligned} \quad (20)$$

Based on the relative degree $r = 2$ (the relative degree for Bergman model is $r = 3$, [30]) the sliding surface S could be considered as follow:

$$S = \left(\frac{d}{dt} + \lambda\right)e_G \quad (21)$$

To ensure that the tracking error e_G converges to zero, the sliding variable and its derivatives must converge to zero. This condition means the remaining on sliding surface S . λ is a positive constant, is defined by frequency bound of unmeasurable uncertainties. $\dot{S}$ can be calculated as follow:

$$\begin{aligned} \dot{S} &= [\ddot{G}(t) - \ddot{G}_{ref}(t)] + \lambda[\dot{G}(t) - \dot{G}_{ref}(t)] \\ &= -K_{xgi} \left[-K_{xi}I(t) + \frac{T_{igmax}}{V_i} f(G(t - \tau_g)) + u(t) \right] G(t) \\ &\quad - K_{xgi}I(t)\dot{G}(t) - \ddot{G}_{ref}(t) + \lambda[\dot{G}(t) - \dot{G}_{ref}(t)] \end{aligned} \quad (22)$$

By considering the input control $u(t)$ as below:

$$\begin{aligned} u(t) &= -\frac{T_{igmax}}{V_i} f(G(t - \tau_g)) + K_{xi}I(t) \\ &+ \frac{[-K_{xgi}I(t)\dot{G}(t) + \ddot{G}(t) + \lambda(\dot{G}(t) - \dot{G}_{ref}(t))]}{(K_{xgi}G(t))} \end{aligned} \quad (23)$$

By substituting the $u(t)$ in (22) it would be seen that $\dot{S} = 0$.

B. Unscented Kalman filter (UKF)

As mentioned, the EKF has more computationally efforts and has some difficulties to implement in the estimation problems [33]. But the UKF has shown the better performance by using the sigma vectors. In contrast of the EKF, no explicit calculation of Jacobian matrix is necessary for the UKF estimation algorithm. The sigma points completely capture both the true mean and covariance of the GRV, and when propagated through the true nonlinear uncertain system, the posterior mean and covariance are accurately obtained [35-36].

Assume $G(n)$ is the n^{th} sample of glucose level at time $t = n$, in measurement equation (15). Consider the state variable $x(n) = G(n)$, has the mean $E[x(n)] = \bar{x}(n)$ and the covariance matrix $\Sigma_x(n/n)$. Essentially, the UKF considers $2L+1$, sigma points where L is the dimension of the state vector. Consider discrete form of the continuous nonlinear function (5) and (15) as follows:

$$G(n+1) = f(G(n), I(n)) \quad (24)$$

$$Z(n) = G(n) + v(n) \quad (25)$$

To derive a recursive algorithm based on unscented transformation there are some definitions as below:

$$\begin{aligned} W_0^m &= \lambda / (L_x + \lambda) \\ W_0^c &= \lambda / (L_x + \lambda) + (1 - \alpha^2 + \beta) \\ W_i^m &= W_i^c = 0.5 / (L_x + \lambda) \quad i = 1, \dots, 2L_x \\ \lambda &= \alpha^2 (L_x + \gamma) - L_x \end{aligned} \quad (26)$$

Where λ and γ are two scaling parameters which usually set to the small positive values. α shows the spread of sigma points around $\bar{G}(n) = \bar{x}(n)$, and β corporates the prior knowledge of $x(n)$ distribution [36]. In order to obtain the mean and covariance $P(n)$, a new sigma matrix $\chi(n)$ consisted of $2L_x + 1$ sigma vectors $\chi_i(n)$ needs to be calculated as below:

$$\begin{aligned} \chi_0(n) &= \bar{x}(n) \\ \chi_i(n) &= \bar{x}(n) + \sqrt{(L_x + \lambda) \Sigma_x(n/n)}_i \quad i = 1, \dots, L_x \\ \chi_i(n) &= \bar{x}(n) - \sqrt{(L_x + \lambda) \Sigma_x(n/n)}_{i-L_x} \quad i = L_x + 1, \dots, 2L_x \\ \chi(n) &= [\chi_0(n) \quad \dots \quad \chi_{2L_x}(n)] \end{aligned} \quad (27)$$

where $\Sigma_x(n/n)_i$ is the i^{th} column of the sigma matrix $\Sigma_x(n/n)$. Consider propagating these sigma vectors through the nonlinear function $f(\cdot)$ in (1),

$$\chi_i(n+1) = f(\chi_i(n)) \quad i = 0, \dots, 2L_x \quad (28)$$

The state variable is estimated by using (16) as below:

$$\hat{G}(t) = -K_{xgi} I(t) G(t) + T_{gh} / V_g \quad (29)$$

The state covariance matrix $\Sigma_x(n/n)$ can be estimated as below:

$$\begin{aligned} \Sigma_x(n+1/n) &= \sum_{i=0}^{2L_x} W_i^c [\chi_i(n+1) - \hat{\chi}(n+1/n)] \\ &\quad \times [\chi_i(n+1) - \hat{\chi}(n+1/n)]^T \end{aligned} \quad (30)$$

Based on (25) the measurement covariance matrix is as follows:

$$\Sigma_z(n+1/n+1) = \Sigma_x(n+1/n+1)$$

Using recursive KF estimator framework for $x(n+1)$,

$$\begin{aligned} \hat{x}(n+1|n+1) &= \hat{x}(n+1|n) + K(n+1) [z(n+1) - \hat{x}(n+1|n)] \\ K(n+1) &= \Sigma_{wz}(n+1/n+1) \Sigma_z(n+1)^{-1} \end{aligned} \quad (31)$$

IV. RESULTS

Different types of fitness functions could be used for validation of particles. In this paper, the mean square error (MSE) and mean absolute error (MAE) are used for glucose reference tracking defined as:

$$MSE(E_G) = [E_G - \bar{E}_G]^T [E_G - \bar{E}_G] \quad (32)$$

$$MAE(E_G) = \text{sum}[E_G - \bar{E}_G] / N$$

Where $E_G = [e_{G1} \ e_{G2} \ \dots \ e_{GN}]^T$ is the glucose reference tracking error for N data points and $e_{Gk} = G(t) - G_{ref}(t)$, $t = kT$, $T = 2 \text{ (min)}$ means the k^{th} glucose reference error and $\text{sum}[E_G - \bar{E}_G] = \sum_{i=1}^N |e_{Gi} - \bar{e}_{Gi}|$. Also, another important criterion for the glucose-insulin controller is the amount of insulin infusion rate well-known as the control effort. Therefore the control effort can be defined as follows:

$$MSE(U) = [U - \bar{U}]^T [U - \bar{U}] \quad (33)$$

$$MAE(U) = \text{sum}[U - \bar{U}] / N$$

Where $U = [u_1 \ u_2 \ \dots \ u_N]^T$ is the insulin infusion rate for N sample times. The best fitness function considers the glucose-insulin controller that balance between reference glucose tracking errors and insulin infusion rates as the control effort:

$$\begin{aligned} MSE(GU) &= [E_G - \bar{E}_G]^T Q [E_G - \bar{E}_G] + [U - \bar{U}]^T R [U - \bar{U}] \\ MAE(GU) &= \text{sum}(Q[E_G - \bar{E}_G]) / N + \text{sum}(R[U - \bar{U}]) / N \end{aligned} \quad (34)$$

Where Q and R are weighted matrices. The numerical values of a T1D patient based on the Palumbo DDE model are used for simulations [12]. The plasma glucose level $G(t)$ is directly measured then the plasma insulin $I(t)$ could be estimated. It must be noted that the simulation results are based on this assumption that plasma insulin value is estimated precisely, i.e. $\hat{I}(t) = I(t)$ and the dynamics of the insulin pump is neglected. Some parameters such as G^* , V_p , γ , V_g , K_{xi} , τ_g , K_{xgi} ,

T_{gh} and T_{igmax} are estimated or calculated according to patient steady state relations. To compare between the proposed HoSM-UKF method performance and the other glucose-insulin regulatory methodologies, the parameters of a typical T1DM patient are used in this paper as shown in Table 1. This selected T1DM patient has a body mass index, BMI=50, and the base BG level is $G_b = G(0) = 12.37 \text{ mM}$ that indicates a higher BG level compared with the normal level. K_{xgi} is the rate of glucose uptake by insulin-dependent tissues per pM of plasma insulin concentration is less than 10^{-4} , that shows a sub-normal insulin delivery rate.

TABLE I. THE NUMERICAL VALUES OF A T1D PATIENT BASED ON THE PALUMBO DDE MODEL [12]

$G_b = 12.37 \text{ mM}$	$I_b = 93.669 \text{ pM}$
$K_{xgi} = (2.51) \cdot 10^{-5} \text{ min}^{-1} \text{ pM}^{-1}$	$G^* = 9 \text{ mM}$
$T_{gh} = 0.003 \text{ min}^{-1} (\text{mmol/kgBW})$	$\tau_g = 24 \text{ min}$
$V_g = 0.187 \text{ L/kgBW}$	$\gamma = 3.205$
$K_{xi} = (1.211) \cdot 10^{-2} \text{ min}^{-1}$	$V_i = 0.25 \text{ L/kgBW}$
$T_{igmax} = 0.242 \text{ min}^{-1} (\text{pmol/kgBW})$	$BMI = 50$

The reference signal of plasma glucose has a decreasing exponentially profile from the value of 12.37 to the final value 4.7 as below:

$$G_{ref}(t) = 4.7 + (12.37 - 4.7) \cdot \exp(-0.01t) \quad (35)$$

Also, Table 2 shows the numerical values are chosen for the insulin actuator and glucose disturbance parameters mentioned in (4) and (7):

TABLE II. THE NUMERICAL VALUES OF INSULIN ACTUATOR & GLUCOSE DISTURBANCES [12]

$a_I = 0.03 \text{ (pM)}$	$w_I = 0.08 \text{ (rad/min)}$
$\beta = -0.02 \text{ (min}^{-1}\text{)}$	$t_m = 160 \text{ (min)}$
$w_G = 0.08 \text{ (rad/min)}$	$\gamma = -0.03 \text{ (mM)}$
$T = 200 \text{ (min)}$	$\delta = 0.012 \text{ (mM)}$
$\alpha = 0.09 \text{ (mM)}$	$a_G = 0.03 \text{ (mM)}$
$t_0 = 350 \text{ (min)}$	

Fig. 1 shows the glucose disturbances profile due to food intake, normal and constant physical activities and emotional disturbances, respectively.

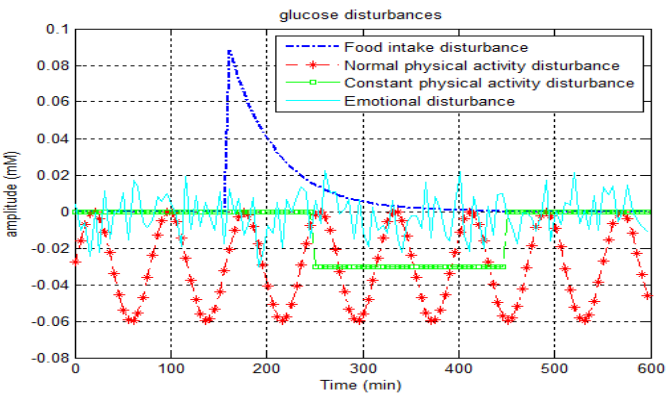


Figure 1. Blood glucose disturbances profile

Fig. 2 shows the stabilization of the patient's BG level into the normal level by using HoSM-UKF approach compared with fuzzy, fuzzy-UKF controllers.

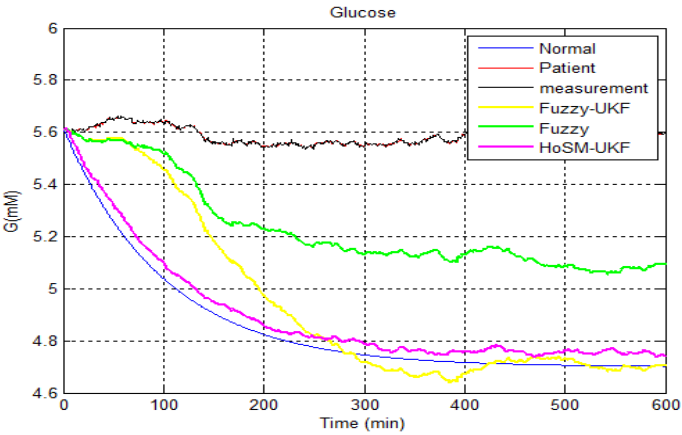


Figure 2. Regulation of plasma glucose by applying HoSM-UKF controller

Fig. 3 illustrates the error obtained from the difference between patient's BG and normal glucose levels in the proposed approach compared with the other control methods.

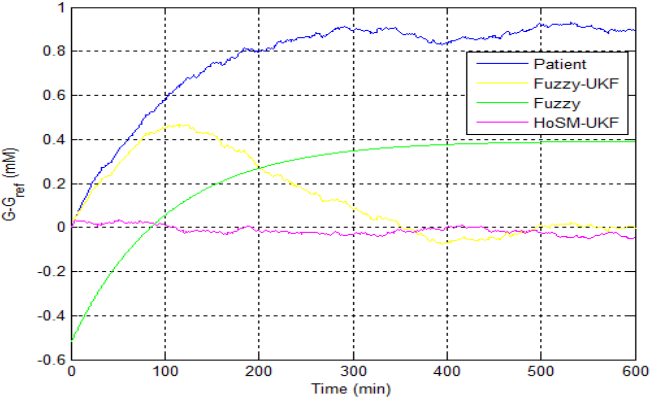


Figure 3. The error of blood glucose tracking respect to the reference glucose

All of the insulin dosing control is based on a $T=2 \text{ (min)}$ glucose sample time. As illustrated in Fig. 3, without applying controller, the BG level remains in the range 5.6 (mM) while by using close-loop controller the BG level gets closer to the normal

or reference amount. The difference between the different controller performances is apparent on finer scale as shown in Fig. 3. The fuzzy controller approach cannot control the blood glucose very well, but by using fuzzy-UKF algorithm, where the noisy measurement is estimated by UKF, the blood glucose can better get to the normal level.

One important aspect of the proposed HoSM-UKF based on UKF estimation is desired BG level tracking while preventing chattering behavior of the insulin infusion rate. This property of the proposed method is a vital feature for T1DM patients. Although the fuzzy and fuzzy-UKF approaches provide excellent or acceptable performances in glucose reference tracking, all of them suffer the chattering phenomena in their insulin infusion rates. Fig. 4 shows the injected insulin infusion rate while preventing chattering by the HoSM-UKF approach, specially over the steady state time interval.

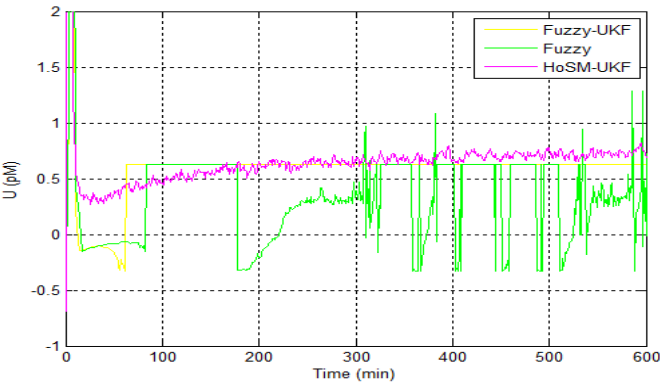


Figure 4. The insulin injection rate by using chattering-free HoSM-UKF controller compare with the other controllers

Fig. 5 indicates the body insulin changes by applying each controllers for this T1DM patient, respectively.

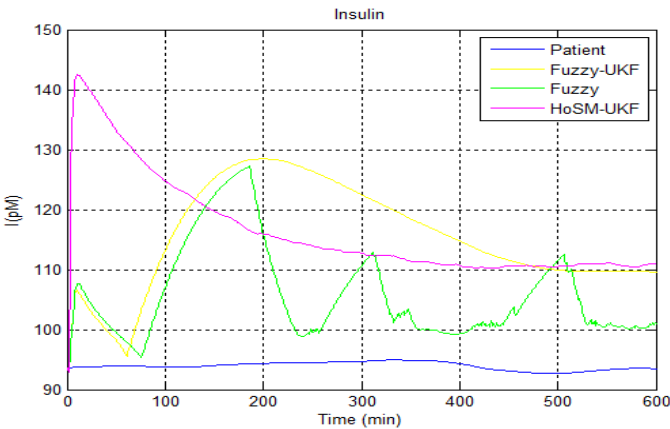


Figure 5. Patient's plasma insulin changes by applying each controllers

In Table 3, the glucose tracking error of each controllers are given and compared with HoSM-UKF controller regarding the MSE and MAE criterions. Although fuzzy and fuzzy-UKF methods are counted as suitable controllers (as seen in Table 3), but this decreasing glucose tracking error would be with a corresponding increment in control effort (as shown in Table 4).

TABLE III. THE CONTROL INPUT EFFORTS OF EACH CONTROLLER

Control method	MSE(G)	MAE(G)
Fuzzy	0.1472	0.3746
Fuzzy-UKF	0.0412	0.1382
HoSM-UKF	0.0066	0.0653

In contrast to fuzzy based controllers, the HoSM controller has a better performance of glucose reference tracking (Table 3) and its control effort generally has an acceptable level and even lower than that of by fuzzy-UKF method (Table 3). Improvement in MSE of the glucose reference tracking for the proposed approach with respect to the fuzzy and fuzzy-UKF approaches is 83.3%, and 53.2%, respectively.

TABLE IV. THE GLUCOSE REGULATORY INPUT EFFORTS

Control method	MSE(U)	MAE(U)
Fuzzy	0.3860	0.2250
Fuzzy-UKF	0.3699	0.5872
HoSM-UKF	0.5796	0.3441

Improvement in MSE of the control effort for the proposed approach with respect to the fuzzy-UKF approach is 41.37%.

As mentioned before, the best criterion should be consider a trade-off between reference glucose tracking error and the control effort was presented in (34). In Table 5, the results of three different methods are summarized regarding the MSE and MAE criterions, Q and R are set to 100 and 1, respectively.

TABLE V. THE GLUCOSE LEVEL CONTROL AND CONTROL INPUT EFFORTS

Control method	MSE(GU)	MAE(GU)
Fuzzy	0.3943	0.7124
Fuzzy-UKF	0.3886	0.8044
HoSM-UKF	0.3144	0.5810

Improvement in MSE for the proposed approach with respect to fuzzy and fuzzy-UKF methodologies is 20.22%, and 18.96%, respectively. And The improvement in MAE for the proposed approach with respect to fuzzy and fuzzy-UKF methods are 18.30%, and 27.5%, respectively.

V. DISCUSSION

T1D patients rely on insulin injection at regular periods of time to maintain the BGC near the normal level. So the ability of smooth insulin delivery is considered as a vital feature of a typical glucose-insulin controller to prevent some of complications such as hypo or hyperglycemia in diabetic patients.

In this paper, a hybrid technique for blood glucose regulation by insulin injections is proposed. One important aspect of the new HoSM-UKF control scheme is the glucose tracking to the desired level as well as the preventing chattering behavior in insulin injection rate. The simulation results shown that the fuzzy and fuzzy-UKF metods suffer the chattering phenomena in their insulin infusion rates. It is shown that the proposed

technique adequately address the issue of reducing chattering phenomena of insulin delivery.

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